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ON THE ONE-DIMENSIONAL EQUATION OF MOTION FOR THE NUMERICAL SIMULATION OF TRANSVERSELY VENTILATED ROAD TUNNELS

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ABSTRACT

The one-dimensional equation of motion for air flow in tunnel ventilation simulation is the subject of the present report. The equation presented by the authors in previous studies was found to be incorrect, and a full derivation of the correct equation of motion for longitudinal air flow in an arbitrary ventilation condition is performed using momentum law. The discrepancy between the two equations is discussed with respect to fluid dynamics and several sample calculations for steady state ventilation are presented in order to demonstrate how the present formulation functions. For each case the velocity in the longitudinal direction and the pressure distribution are illustrated. The first three examples showed good agreement with the literature, whereas the fourth and fifth examples included an imbalance-transverse system, which can be calculated only by the present method, and no comparison with the previous studies was possible.

NOMENCLATURE

A : cross sectional area of tunnel [m^2]
A_m : equivalent drag area of vehicles [m^2]
F_f : force due to tunnel friction [N]
F_t : force due to traffic [N]
ℓ : tunnel length or length of a ventilation section [m]
N : traffic density [veh/h]
n : equivalent number of vehicles per unit length [veh/m]
p : pressure [Pa]
p_t : driving pressure by traffic [Pa]
Q : air flow rate ($=AV$) [m^3/s]
q_b : flow rate of the fresh air supply per unit length [m^3/s/m]
q_e : exhaust flow rate per unit length [m^3/s/m]
t: time [s]
V : longitudinal air flow velocity [m/s]
V_t : average traffic speed [m/s]
x: longitudinal coordinate [m]
λ : tunnel friction coefficient [-]
ξ_e : coefficient of exhausting momentum [-]
ξ_b : coefficient of blowing momentum [-]
ρ : air density [kg/m^3]
ϕ : velocity gradient [1/s]

SUBSCRIPTS

$_+$: positive direction
\- : negative direction
$_0$:value at the beginning of a ventilation section

INTRODUCTION

Road tunnels are ventilated either by a transverse or a longitudinal ventilation system. The former requires ventilation ducts that are separate from the traffic room, whereas the latter uses the traffic room itself as the ventilation duct. The necessity for complex new ventilation systems is becoming more common in order to maintain the environment both inside and outside tunnels. To deal with these new ventilation systems, it is essential to establish the generalized equation of motion for the air flow inside the tunnel. Since the beginning of the Kan-etsu tunnel project, numerical methods have been used extensively for longitudinally ventilated tunnels [1],[2]. However, numerical simulation has been less actively developed for transverse ventilation, probably because of the idea that the analysis of the longitudinal air flow was not important in the transverse system even for the fire case. The authors have pointed out the necessity of analyzing the longitudinal air flow in the transverse ventilation system, especially for the purpose of emergency operations, and have tried to establish the basic methodology of numerical simulation [3],[4]. In the one of these studies [4], however, the basic equation of motion was found to contain an error, which is corrected in the present study. Thus, in the present study, the authors attempt to establish a more generalized equation of motion for the longitudinal air flow under various ventilation conditions (systems) and present exemplary calculations in order to show the workings of the proposed equation.

CONVENTIONAL FORM OF ACCELERATION TERM

In the previous studies [3],[4] by the present authors, the following equation of motion was used for the analysis of transversely ventilated tunnels:

$$dm\left(\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}\right)=-\frac{\partial p}{\partial x}dxA+dF_t+dF_f\ , \tag{1}$$

where dm is the mass in the infinitesimal control volume, V is the longitudinal air flow velocity, p is the pressure, A is the cross sectional area of the traffic room, x and t are the spatial and temporal coordinates, respectively, and dF_t and dF_f the traffic force and the wall friction force, respectively. The terms on the right-hand side (RHS) are the forces imposed on the air in the control volume. The first term in the parentheses in the left-hand side (LHS) of Eq. (1) is the local acceleration, and the second is the convective acceleration, both of which are very commonly used in the basic fluid dynamics equations and therefore would likely contain no errors.

Equation (1) is integrated along the x axis in order to obtain an ordinary differential equation of the velocity at the entrance with regard to time. The integration must be performed carefully, because the functions dF_t and dF_f can change sign in the range being considered. The integration will be discussed in detail in a later section as well as in the appendix.

FORMULATION IN TERMS OF MOMENTUM LAW

Further consideration of the longitudinal air flow revealed that Eq. (1) might be incorrect. Therefore, the present authors attempt to reformulate the one-dimensional equation of motion for longitudinal air flow based on momentum law.

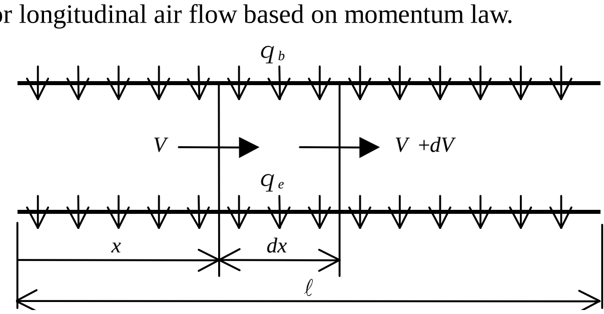


Fig. 1: Momentum law for an infinitesimal region

The momentum law

(outflow momentum) - (inflow momentum)
=(total force) - (mass * acceleration) (2)

is applied in the following manner. The inflow momentum into the control volume with an infinitesimal length dx (as shown in Fig. 1) is the summation of the momentums from upstream of the main duct and that of the fresh air supply,

$$M_{in} = \rho V\left|V\right|A+\xi_b\rho Vq_b dx\ ,$$

Similarly, the outflow momentum is

$$M_{out} = \rho\left(V+dV\right)\left|V+dV\right|A+\xi_e\rho Vq_e dx\ ,$$

where ρ is the density of air, ξ_b and ξ_e are the coefficients representing the rate of the x-components of the momentums of the inflow and outflow, respectively, and q_b and q_e are the flow rate of blowing fresh air and that of exhaustion per unit length, respectively. The continuity equation is

$$V=\frac{Q_0-x\left(q_b-q_e\right)}{A}=V_0+\frac{x\left(q_b-q_e\right)}{A}. \tag{3}$$

The total force can be categorized into the following three forces:

F_{cs} : force from the control surface $\quad pA-(p+dp)A$

F_r : reaction force from solid surface $\quad -\lambda\dfrac{dx}{D}\dfrac{\rho}{2}AV\,|V\,|$

F_m : mass force $\quad dF_t$

Therefore, the momentum law is

$$F_{cs}+F_r+F_m=\rho Adx\frac{\partial V}{\partial t}+M_{out}-M_{in}\,. \tag{4}$$

Each term can be substituted for to obtain

$$-dpA-\lambda\frac{dx}{D}\frac{\rho}{2}AV\,|V\,|+dF_t=\rho A\frac{\partial V}{\partial t}dx$$
$$+\rho A\left(V+dV\right)|V+dV\,|+\xi_e\rho Vq_e dx-\rho AV\,|V\,|-\xi_b\rho Vq_b dx.$$

By neglecting the higher-order terms, the generalized form of the equation of motion is obtained as

$$\frac{\partial p}{\partial x}=-\rho\left\{\frac{\partial V}{\partial t}+\frac{\lambda}{2D}V\,|V\,|+2V\frac{\partial V}{\partial x}+\frac{\xi_b q_b-\xi_e q_e}{A}V\right\}+\frac{1}{A}\frac{\partial F_t}{dx}. \tag{5}$$

In inducing this expression, M_{out} and M_{in} are exchanged when V<0.

REASON FOR THE DISCREPANCY

In the numerical simulation for transversely ventilated tunnels, the authors had been used Eq. (1) as the basic equation of motion [3],[4]. Further consideration, however, revealed that Eq. (1) might be incorrect, and the authors tried to reformulate the equation of motion based on momentum law. At the time, the idea that errors were present in Eq. (1) was almost inconceivable, because the convective term of the equation is one of the most common expressions used in fluid dynamics. However, let us here discuss the reason for the discrepancy between Eqs. (1) and (5).

One difference between Eqs. (1) and (5) is that the convective acceleration of Eq. (5) is twice that of Eq. (1). This phenomenon is generally referred to as the “air curtain effect”. Another difference is that the final term in parenthesis in Eq. (5) represents the momentum in and out caused by the fresh air and the exhaustion ventilation.

Since the fresh air is usually blown into the traffic room from the duct without a momentum component in the axial direction, the main stream must be responsible for the acceleration of the fresh air, so that the newly introduced air has the same longitudinal velocity as the surrounding fluid. This means that the main flow must accelerate both the flowing air itself (when fresh air is introduced, the main flow must be accelerated due to continuity law) and the new fresh air. This is considered to be the primary reason for the difference between the two expressions.

INTEGRATION ALONG THE TUNNEL AXIS

Although the equation of motion developed in the previous section is applicable for the purpose of numerical simulation in the unsteady state, in this section, the authors present the integration along the tunnel axis under the steady-state condition.

Supposing that the flow rate of the fresh air supply and that of the exhaustion per unit length of the tunnel are q_b and q_e, respectively, which are constant along a ventilation section, the continuity law reads:

$$V = V_0 + \frac{q_b - q_e}{A} x = V_0 + \phi x \quad , \quad \frac{dV}{dx} = \frac{q_b - q_e}{A} = \phi \, , \qquad (3)'$$

where the subscript 0 denotes the value at the beginning of the ventilation section and ϕ is the velocity gradient. The expression “ventilation section” denotes the range of x, over which q_b and q_e remain constant. The integration of the pressure gradient over the entire ventilation section is equivalent to the pressure difference between both ends of the section,

$$\int_0^{\ell} p dx = p_1 - p_0 \, ,$$

which is normally related to the natural wind condition, and may be considered to be known.

In the following, we assume the coefficients are as follows. The momentum brought into the traffic room by the fresh air is assumed to be zero; thus, ξ_b = 0. Whereas the exhaust air is supposed to flow out with the momentum based on the mean velocity, yielding ξ_e = 1. Thus, the steady-state equation of motion can be written as

$$\frac{dp}{dx} = -\rho \left\{ \frac{\lambda}{D} V |V| + 2V \frac{dV}{dx} + \frac{q_e}{A} V \right\} + \frac{1}{A} \frac{dF_t}{dx} \, . \qquad (6)$$

Substituting (3)’ into V’s in Eq. (6), the pressure gradient is expressed as a function of x as follows:

$$\frac{dp}{dx} = -\rho \left\{ \frac{\lambda}{2D} (V_0 + \phi x) | V_0 + \phi x | + \left(2\phi + \frac{1}{A} q_e \right) (V_0 + \phi x) \right\} + \frac{1}{A} \frac{dF_t}{dx} \qquad (7)$$

Special attention must be exercised in integrating this formula, because the function to be integrated includes absolute values, which causes the sign to vary according to the region. The results are as follows:

(a) For $V_0 < 0$, $V_0 + \phi\ell \geq 0$,

$$\int_0^{\ell} \frac{dp}{dx} dx = -\rho \left\{ \frac{\lambda}{2D} \left(\frac{2}{3} \frac{V_0^3}{\phi} + V_0^2 \ell + V_0 \phi \ell^2 + \frac{1}{3} \phi^2 \ell^3 \right) + \ell \left(2\phi + \frac{q_e}{A} \right) \left(V_0 + \frac{\phi}{2} \ell \right) \right\} + \frac{F_t}{A} . \qquad (8)$$

(b) For $V_0 \geq 0$, $V_0 - \phi\ell < 0$,

$$\int_0^{\ell} \frac{dp}{dx} dx = \rho \left\{ \frac{\lambda}{2D} \left(\frac{2}{3} \frac{V_0^3}{\phi} + V_0^2 \ell + V_0 \phi \ell^2 + \frac{1}{3} \phi^2 \ell^3 \right) + \ell \left(2\phi + \frac{q_e}{A} \right) \left(V_0 + \frac{\phi}{2} \ell \right) \right\} + \frac{F_t}{A} . \qquad (9)$$

(c) For $V_0 \geq 0$, $V_0 + \phi\ell \geq 0$,

$$\int_0^{\ell} \frac{dP}{dx} dx = -\rho \left\{ \frac{\lambda}{2D} \left(V_0^2 \ell + V_0 \phi \ell^2 + \frac{1}{3} \phi^2 \ell^3 \right) + \ell \left(2\phi + \frac{q_e}{A} \right) \left(V_0 + \frac{\phi}{2} \ell \right) \right\} + \frac{F_t}{A} . \qquad (10)$$

(d) For $V_0 < 0$, $V_0 - \phi\ell < 0$,

$$\int_0^{\ell} \frac{dp}{dx} dx = \rho \left\{ \frac{\lambda}{2D} \left(V_0^2 \ell + V_0 \phi \ell^2 + \frac{1}{3} \phi^2 \ell^3 \right) + \ell \left(2\phi + \frac{q_e}{A} \right) \left(V_0 + \frac{\phi}{2} \ell \right) \right\} + \frac{F_t}{A} . \qquad (11)$$

When the tunnel has multiple ventilation sections, these sections can be connected under continuous pressure and the flow rate conditions. The force induced by traffic F_t is described in the Appendix.

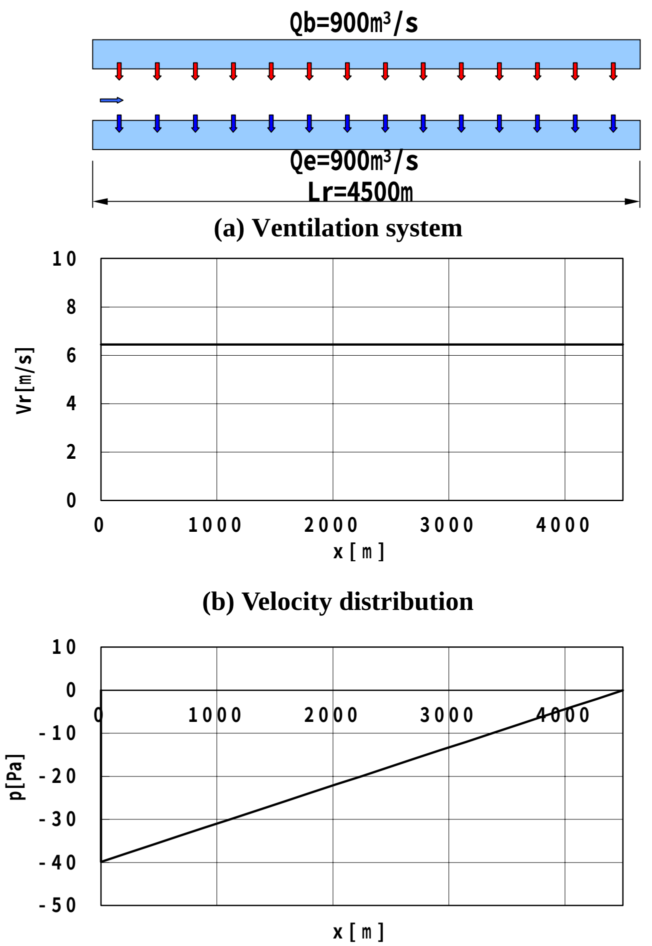
EXAMPLE CALCULATIONS

Example calculations are performed for several steady-state conditions. Although the equation of motion formulated here is in generalized form, and thus can be applied to unsteady-state problems, for simplicity, the present study deals exclusively with steady-state problems. Through steady-state calculation, the balance between the convective term and the external force term can be confirmed quantitatively. For each of five examples, figures are presented which illustrate the ventilation system, velocity distribution and pressure distribution. Each of the five examples have the following common parameters: tunnel length; 4,500 m, cross sectional area; 42 m^2, mean projection area of the vehicles; 2.0 m^2, traffic rate; 3,000 vehicles per hour (one-way traffic), vehicle speed; 60 km/h, wall friction coefficient; 0.025, and entrance loss coefficient; 0.6. The ventilation air flow rate is 0.2 m^3/s unless otherwise specified.

Example 1: Full transverse system. Q_b=900 m^3/s is supplied uniformly along the whole tunnel, and the same quantity, Q_e=900 m^3/s, is exhausted. The longitudinal velocity is constant (6.44 m/s), and the pressure distribution is linear. (Fig. 2)

Example 2: Semi-transverse ventilation system. Q_b=900 m^3/s of exclusively fresh air is supplied along the entire tunnel. In this system, the longitudinal air velocity becomes linear, and the pressure distribution is indicated by a quadratic function (Fig. 3).

Example 3: Combined ventilation system. The first half of the tunnel consists of a full transverse system (Q_b=450 m^3/s, Q_e=450 m^3/s), and the remainder consists of a semi-transverse system with exhaustion (Q_b=0 m^3/s, Q_e=450 m^3/s). This type of ventilation system is appropriate for tunnels, from which exhaust gas cannot be vented due to environmental restrictions. The velocity distribution bends at the center, and the pressure distribution also changes at the center. (Fig. 4)

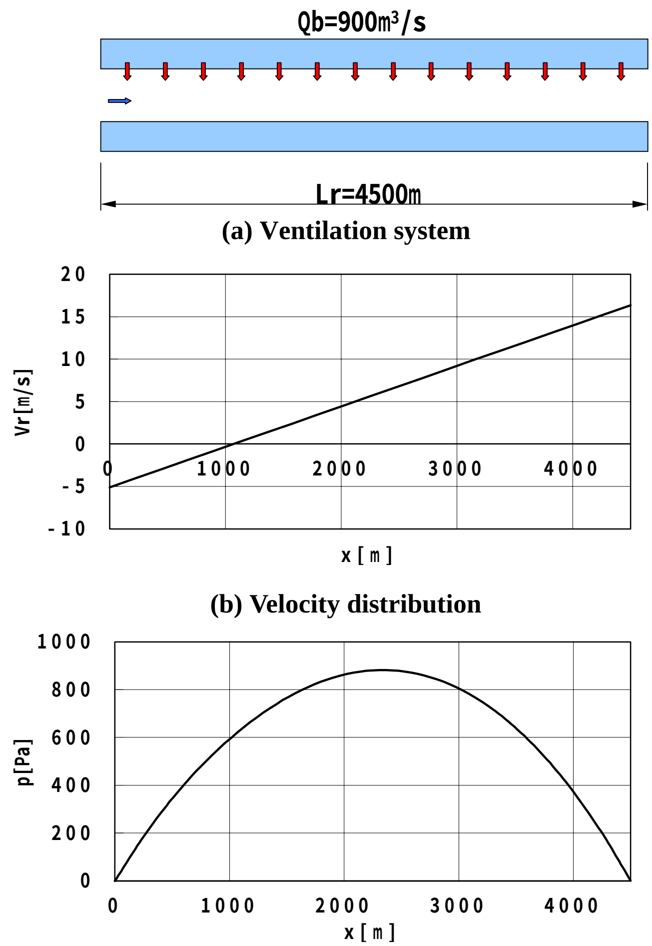


(a) Ventilation system

(b) Velocity distribution

(c) Pressure distribution

Fig: 2 Full transverse ventilation system (Example 1)



(a) Ventilation system

(b) Velocity distribution

(c) Pressure distribution

Fig: 3 Semi-transverse ventilation with fresh air supply (Example 2)

Example 4: Imbalance-transverse ventilation system. The fresh air supply is Q_b=450 m^3/s, and the exhaustion is Q_e=900 m^3/s. This system may also provides environmentally friendly portal exhaustion. Excessive exhaustion causes a higher inlet velocity and a lower outlet velocity. The pressure distribution is asymmetric due to traffic. (Fig. 5)

Example 5: Imbalance-transverse ventilation system. The fresh air supply is Q_b=900 m^3/s, and the exhaustion is Q_e=450 m^3/s. In this system, the fresh air supply is larger, and the air flow is accelerated. The maximum pressure of the tunnel is located near the center of the tunnel because the inlet velocity is small. In addition, entrance loss is not observable in the figure. (Fig. 6)

The results of the first three examples were compared to those presented in Reference [6], and the results of the present study were confirmed to be valid. Examples 4 and 5, which include an imbalance-transverse system, were calculated for the first time using the proposed formulation.

CONCLUSION

The formulation of the equation of motion for longitudinal air flow in tunnel ventilation was discussed and example calculations were presented. In the present study:

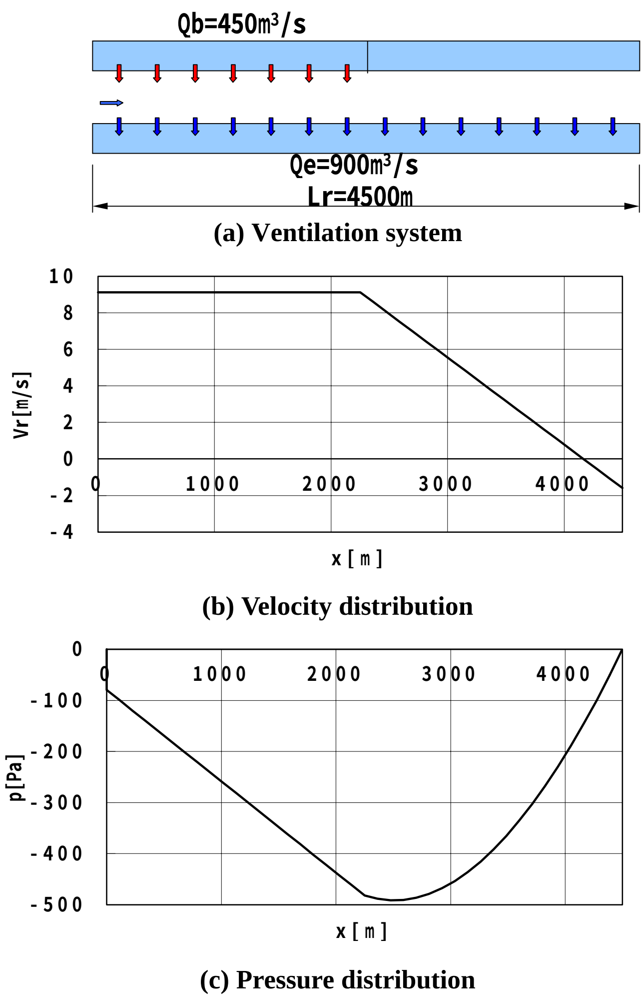


Fig: 4 Combined ventilation system (Example 3)

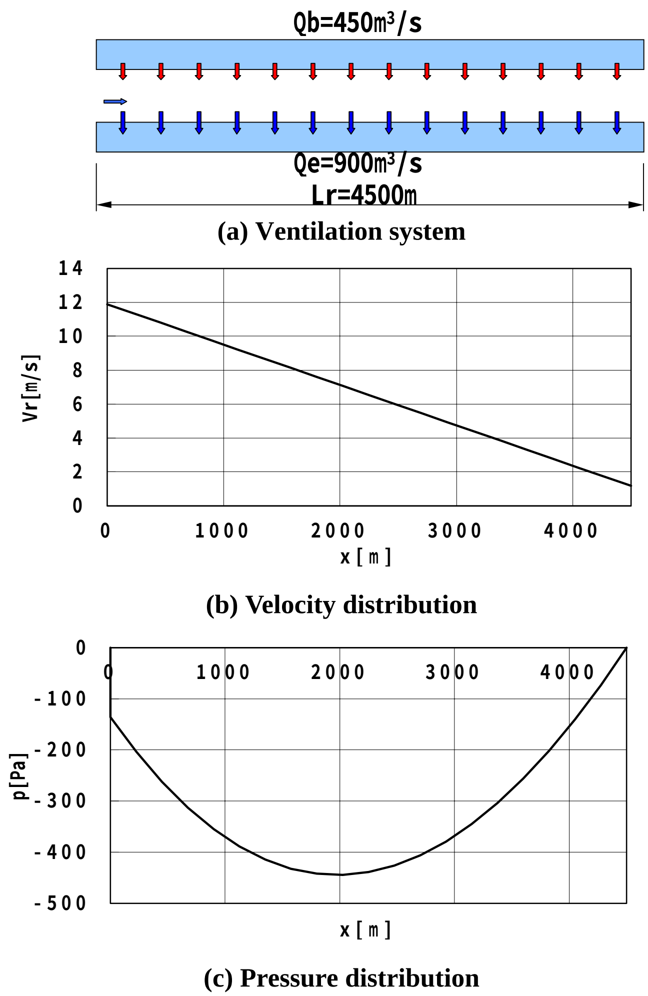
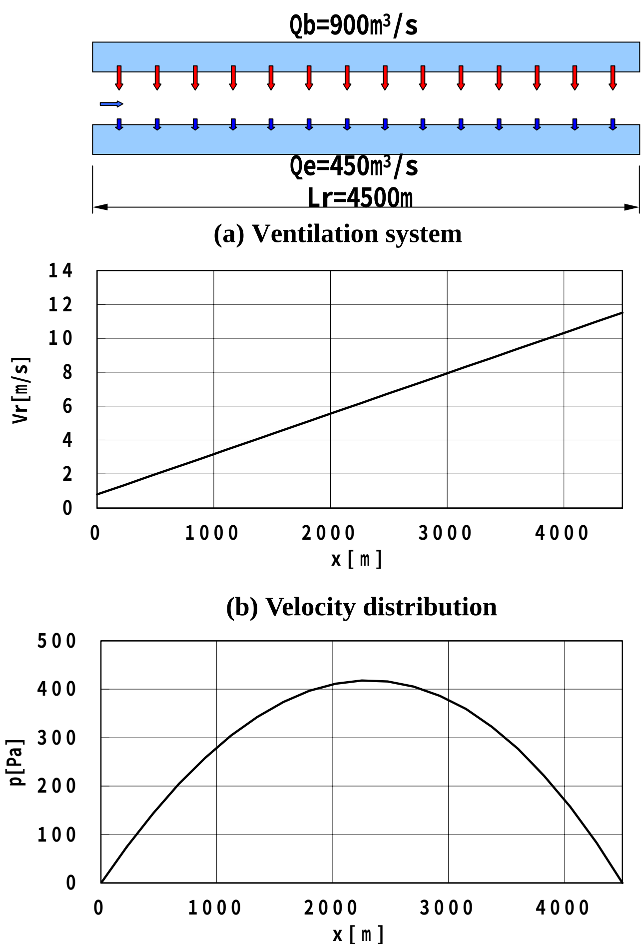


Fig: 5 Imbalance-transverse ventilation system (Example 4 - larger exhaustion)

1) The expression of the one-dimensional equation of motion for tunnel ventilation air flow is formulated and presented. The proposed equation can be used to analyze both steady-state and unsteady-state conditions.
2) The validity of the current formulation is confirmed through several example calculations.
3) The fallibility of applying a governing equation is exemplified, and the importance of considering phenomena from a more basic viewpoint in order to model mathematically the problem properly is shown.

Confirmation of the present results appears to be rather difficult in terms of comparison with previous experimental results. However, the present authors hope that the above discussion will be verified through a variety of indirect measurements performed at actual tunnels.



(a) Ventilation system

(b) Velocity distribution

(c) Pressure distribution

Fig: 6 Imbalance-transverse ventilation system (Example 5 - larger fresh air supply)

REFERENCES

[1] Ohashi, H., Mizuno, A., Nakahori, I. and Ueki, M., "A new ventilation method for the Kan-etsu road tunnel", Proc. 4th International Symposium on the Aerodynamics and Ventilation of Vehicle Tunnels (York, U.K., Mar. 23-25, 1982), BHRA, pp.31-47.

[2] Mizuno, A., Ohashi, H., Nakahori, I. and Okubo, N., Proc.5th International Symposium on the Aerodynamics and Ventilation of Vehicle tunnels (Lille, France, May 20-22, 1985), BHRA 1985, pp.77-91.

[3] Mizuno, A. and Ichikawa, A., Proc. 7th International Symposium on Aerodynamics and Ventilation of Vehicle tunnels (Brighton, UK, 27-29 November 1991), BHRA, pp349-364.

[4] Mizuno, A. and Ichikawa, A., Safety in Road and Rail Tunnels (First International Conference, Basel, Switzerland, 23-25 November 1992), ITC, pp425-438.

[5] Ibukiyama, S., "Road tunnel ventilation", Riko-tosho publishing, (1962). (in Japanese)

[6] "Report on the study of combined ventilation system", Express Highway Research Foundation of Japan, 1971. (in Japanese)

APPENDIX

The force induced by the traffic in Eqs. (8) through (11) is calculated by the integration of the local force along the whole tunnel section.

$$F_t = \int_0^{\ell} dF_t \ ,$$

where

$$dF_t = dp_{t+}A + dp_{t-}A ,$$

in which the first and second term on the RHS arise from the traffic force in the positive and negative directions, respectively. The pressure gain due to traffic in the positive direction over an infinitesimal region dx is

$$dp_{t+} = \frac{\rho}{2}\frac{A_m}{A} n_+ \left(V_t - V\right) | V_t - V \mid dx \ , \qquad (12)$$

where V is the local air flow velocity and n_+ is the traffc density expressed in vehicles per unit length [veh./m],

$n_+ = \left| N_+ / 3600 V_t \right|$.

By defining $\psi_+ = V_{t+} - V_0$, and using the continuity,

$$dp_{t+} = \frac{\rho}{2}\frac{A_m}{A} n_+ \left(\psi_+ - \phi x\right) | \psi_+ - \phi x \mid dx \ .$$

This expression is quadratic with regard to x and can be integrated along the tunnel division through special consideration for the treatment of the absolute value. The results are:

For $\psi_+ \ge 0$, $\psi_+ - \phi \ell \ge 0$,

$$\int_0^{\ell} \frac{dp_{t+}}{dx} dx = \frac{\rho}{2} \frac{A_m}{A} n_+ \left(\psi_+{}^2 - \psi_+ \phi \ell + \frac{\phi^2}{3} \ell^3 \right). \tag{13}$$

For $\psi_+ < 0$, $\psi_+ - \phi\ell < 0$,

$$\int_0^{\ell} \frac{dp_{t+}}{dx} dx = -\frac{\rho}{2} \frac{A\ell}{A} n_+ \left(\psi_+{}^2 - \psi_+ \phi \ell + \frac{\phi^2}{3} \ell^3 \right). \tag{14}$$

For $\psi_+ \geq 0$, $\psi_+ - \phi\ell < 0$,

$$\int_0^{\ell} \frac{dp_{t+}}{dx} dx = -\frac{\rho}{2} \frac{A_m}{A} n_+ \ell \left(-\frac{2}{3} \frac{\psi_+{}^3}{\phi^3} + \psi_+{}^2 \ell - \psi_+ \phi \ell + \frac{\phi^2 \ell^2}{3} \right). \tag{15}$$

For $\psi_+ < 0$, $\psi_+ - \phi\ell \geq 0$,

$$\int_0^{\ell} \frac{dp_{t+}}{dx} dx = \frac{\rho}{2} \frac{A_m}{A} n_+ \ell \left(-\frac{2}{3} \frac{\psi_+{}^3}{\phi^3} + \psi_+{}^2 \ell - \psi_+ \phi \ell + \frac{\phi^2 \ell^2}{3} \right). \tag{16}$$

In the same manner, the traffic force in the opposite (negative) direction can be integrated using the definition $\psi_- = V_{t-} - V_0$ as follows. It should be noted that V_{t-} takes a negative value.

For $\psi_- \geq 0$, $\psi_- - \phi\ell \geq 0$,

$$\int_0^{\ell} \frac{dp_{t-}}{dx} dx = \frac{\rho}{2} \frac{A_m}{A} n_- \left(\psi_-{}^2 - \psi_- \phi \ell + \frac{\phi^2}{3} \ell^3 \right). \tag{17}$$

For $\psi_- < 0$, $\psi_- - \phi\ell < 0$,

$$\int_0^{\ell} \frac{dp_{t-}}{dx} dx = -\frac{\rho}{2} \frac{A_m}{A} n_- \left(\psi_-{}^2 - \psi_- \phi \ell + \frac{\phi^2}{3} \ell^3 \right). \tag{18}$$

For $\psi_- \geq 0$, $\psi_- - \phi\ell < 0$,

$$\int_0^{\ell} \frac{dp_{t-}}{dx} dx = -\frac{\rho}{2} \frac{A_m}{A} n_- \ell \left(-\frac{2}{3} \frac{\psi_-{}^3}{\phi^3} + \psi_-{}^2 \ell - \psi_- \phi \ell + \frac{\phi^2 \ell^2}{3} \right). \tag{19}$$

And for $\psi_- < 0$, $\psi_- - \phi\ell \geq 0$,

$$\int_0^{\ell} \frac{dp_{t-}}{dx} dx = \frac{\rho}{2} \frac{A_m}{A} n_- \ell \left(-\frac{2}{3} \frac{\psi_-{}^3}{\phi^3} + \psi_-{}^2 \ell - \psi_- \phi \ell + \frac{\phi^2 \ell^2}{3} \right). \tag{20}$$