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表紙

$H^*(S^\infty \times_\pi X^2)$, $H_*(S^\infty \times_\pi X^2)$ の構造

～ $H_*(W \otimes_\pi H_*(X)^{(2)})$ と $H^*(\mathrm{Hom}_\pi(W, H^*(X)^{(2)}))$ における cup product と cap product の確定と証明を中心として～

H3年10月15日 ～ H7年8月12日 まとめ

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参考文献

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Acyclic Model の方法

abel群の圏を $\mathcal{G}$　$\partial\mathcal{G}$ chain complex の圏

Def 1. 共変函手 C

圏 $\mathcal{G}$ から圏 $\mathcal{G}'$ への共変函手とは、

$\mathcal{G}$ の対象 X に $\mathcal{G}'$ の対象 $F(X)$ を
$\mathcal{G}$ の射 $f: X \to Y$ に $\mathcal{G}'$ の射 $F(f): F(X) \to F(Y)$ を対応させるもので

$F(g \circ f) = F(g) \circ F(f)$
$F(1_x) = 1_{F(x)}$　なる関係を満たす。

Def 2. 次元 n で非輪状なモデル

圏 $\mathcal{A}$ given　$\mathcal{M} \subset \mathcal{A}$ かつ $\mathcal{M}$ is a set
$\mathcal{A}$ は モデル $\mathcal{M}$ を持つ圏という。この時 $\mathcal{A}$ から $\partial\mathcal{G}$ への共変函手 C は 各 $Y \in \mathcal{M}$ に対して
$H_n(C(Y)) = 0$ を満たすならば、次元 n で非輪状であるという。

Def 3. 表現可能 (representable)

F is a covariant from a category $\mathcal{A}$ with a model $\mathcal{M}$ to an abelian group category $\mathcal{G}$.

$$F: \mathcal{A} \to \mathcal{G}$$
$$F(g \circ f) = F(g) \circ F(f)$$
$$F(1_x) = 1_{F(x)}$$

$$\hat{F}: \mathcal{A} \to \mathcal{G}$$

$\overset{\cup}{X} \mapsto (u, c)$ の全体をBaseとする自由Abel群
$u: M \to X$.　$\mathcal{A}$ の射　$M \in \mathcal{M}$　$X \in \mathcal{A}$.
$c \in F(M)$

$\mathcal{A}$ の射 $f: X \to Y$ に対して
$\hat{F}(f): \hat{F}(X) \to \hat{F}(Y)$
$\hat{F}(f)(u, c) = (f \circ u, c)$ で定義される準同型写像とする

Def 3 の続き

この時，自然な変換 $\rho : \hat{F} \to F$ を

$$\rho(X)(u,c) = F(u)c \qquad (u,c) \in \hat{F}(X)$$

により定義．

$X, Y \in \mathcal{A} \quad f : X \to Y$

$\iota \quad F \to \hat{F}$

$\rho \circ \iota = 1$ が成り立つ時 表現可能という．

ι を F の表現という．

Def 4 自然な変換 (natural transformation)

$T_1, T_2 \quad \mathcal{C} \to \mathcal{D} \qquad T_1\ T_2$ Functor

$$\begin{tikzcd}
T_1(X) \arrow[r, "T_1(f)"] \arrow[d, "\varphi(X)"'] & T_1(Y) \arrow[d, "\varphi(Y)"] \\
T_2(X) \arrow[r, "T_2(f)"'] & T_2(Y)
\end{tikzcd}$$

T_1, T_2 covariant $\qquad \varphi$ is a natural transformation.

例 位相空間の圏 $\mathcal{A}$.

chain complex の圏 $\partial\mathcal{G}$. $\qquad \mathcal{G}$ は可換群の圏

モデル $\mathcal{M}$ を $\{\Delta^n\}$ $(n=0,1,2\cdots)$ 全体とする．

$\hat{F}(X)$ は対 (u,c) $\qquad u : \Delta^n \to X$ $\mathcal{A}$ の写射

$c \in C_n(\Delta^n)$

$c \quad \Delta^n \to \Delta^n$.

$\rho : \hat{F} \to F$

$\rho(X)(u,c) = F(u)c. \qquad f : X \to Y \quad F(f)(\rho(X)(u,c)) = F(f)\circ(F(u)c)$

$= F(f\circ u)c = \rho(X)(f\circ u, c)$

$= \rho(X)(\hat{F}(f)(u,c))$

$\iota : F \to \hat{F}$.

$\iota(X)(\sigma) = (\sigma, 1_n)$

$\iota(X) = C_n(X)$

$\sigma \in C_n(X)$

$\rho\circ\iota = \rho\circ\iota(\sigma) = \rho\circ(\sigma, 1_n) = \sigma$. $\qquad \therefore$ 表現可能

定理 Let $\mathcal{A}$ be a category with models $\mathcal{M}$ and let C and C' be covariant functors from $\mathcal{A}$ to the category of chain complexes such that C is free and nonnegative and C' is acyclic in positive dimensions. Then

(a) Any natural transformation $H_0(C) \to H_0(C')$ is induced by a natural chain map $\tau : C \to C'$.

(b) Two natural chain maps $\tau, \tau' : C \to C'$ inducing the same natural transformation $H_0(C) \to H_0(C')$ are naturally chain homotopic

proof. For every object X of $\mathcal{A}$ we must define a chain map $\tau(X) : C(X) \to C'(X)$ [or a chain homotopy $D(X) : \tau(X) \simeq \tau'(X)$] such that if $h : X \to Y$ is a morphism in $\mathcal{A}$, then

$$\tau(Y)C(h) = C'(h)\tau(X) \qquad [\text{or } D(Y)C(h) = C'(h)D(X)]$$

For $q \geq 0$ let $\{g_j \in C_q(M_j)\}_{j \in J_q}$ be a basis for C_q, where $M_j \in \mathcal{M}$ for each $j \in J_q$. Then $C_q(X)$ has the basis

$$\{C_q(f)(g_j)\}_{j \in J_q,\ f \in \hom(M_j, X)}$$

It follows that $\tau_q(X)$ [or $D_q(X)$] is determined by the collection $\{\tau_q(M_j)(g_j)\}_{j \in J_q}$ and the equation

(a) $$\tau_q(X)\left(\sum n_{ij} C_q(f_{ij})(g_j)\right) = \sum n_{ij} C'_q(f_{ij}) \tau_q(M_j)(g_j)$$

Free ゆえ.

or by the collection $\{D_q(M_j)g_j\}_{j \in J_q}$ and the equation

(b) $$D_q(X)\left(\sum n_{ij} C_q(f_{ij})(g_j)\right) = \sum n_{ij} C'_{q+1}(f_{ij}) D_q(M_j)(g_j)$$

We shall define $\tau_q(X)$ by induction on q so that

(c) $$\partial \tau_q(X) = \tau_{q-1}(X)\partial$$

and define $D_q(X)$ by induction on q so that

(d) $$\partial D_q(X) = \tau_q(X) - \tau'_q(X) - D_{q-1}(X)\partial$$

Having defined τ_i [or D_i] for $i<q$ where $q>0$, it suffices to define $\tau_q(M_j)(g_j)$ for $j \in J_q$ so that :

(e) $$\partial \tau_q(M_j)(g_j) = \tau_{q-1}(M_j)(\partial g_j)$$

$$g_j$$

$$\begin{tikzcd} C_q(M_j) \arrow[r, "\partial"] \arrow[d, "\tau_q(M_j)"] & C_{q-1}(M_j) \arrow[d, "\tau_{q-1}(M_j)"] \\ C'_q(M_j) \arrow[r, "\partial"] & C'_{q-1}(M_j) \end{tikzcd}$$

and to define $D_q(M_j)(g_j)$ for $j \in J_q$ so that

(f) $$\partial D_q(M_j)(g_j) = \tau_q(M_j)(g_j) - \tau'_q(M_j)(g_j) - D_{q-1}(M_j)(\partial g_j)$$

since $\tau_q(X)$ [and $D_q(X)$] are then determined by equation (a) [or by (b)]. It will then be true that $\tau_q(X)$ [and $D_q(X)$] are natural and will satisfy equation (c) [and (d)].

Given a natural transformation $\varphi : H_0(C) \to H_0(C')$, the inductive definition of τ proceeds as follows. For $q=0$, we define $\tau_0(M_j)(g_j)$ for $j \in J_0$ to be any element of $C'_0(M_j)$ such that $\{\tau_0(M_j)(g_j)\} = \varphi(M_j)\{g_j\}$. (by nonnegative)

We use equation (a) to define $\tau_0(X)$ for all X. Then, for $g \in C_0(X)$, $\{\tau_0(X)(g)\} = \varphi(X)\{g\}$. In particular, for $j \in J_1$, $\tau_0(M_j)(\partial g_j)$ is a boundary in $C'_0(M_j)$. Hence we can define $\tau_1(M_j)(g_j) \in C'_1(M_j)$ so that $\partial \tau_1(M_j)(g_j) = \tau_0(M_j)(\partial g_j)$. We then use equation (a) to define $\tau_1(X)$ for all X. Assuming τ_i defined for $i<q$, where $q>1$, so that equation (c) is satisfied, we observe that the right-hand side of equation (e) is a cycle of $C'_{q-1}(M_j)$. Because $q>1$, $H_{q-1}(C'(M_j))=0$, and we define $\tau_q(M_j)(g_j)$ to satisfy equation (e). We next define $\tau_q(X)$ for all X to satisfy equation (a).

This completes the definition τ.

Given $\tau, \tau' : C \to C'$ such that τ and τ' induce the same natural transformation $H_0(C) \to H_0(C')$, we define $D_0(M_j)(g_j)$ for $j \in J_0$ to be any element of $C'_1(M_j)$ whose boundary equals $\tau_0(M_j)(g_j) - \tau'_0(M_j)(g_j)$. Then $D_0(X)$ is defined for all X by equation (b). Assuming D_i defined for $i<q$, where $q>0$, so that equation (d) is satisfied, we observe that the right-hand side of equation (f) is a cycle of $C'_q(M_j)$.

$$\partial \tau_q(M_j)(g_j) - \partial \tau_q'(M_j)(g_j) - \partial D_{q-1}(M_j)(\partial g_j)$$

Because $q>0$, $H_q(C'(M_j))=0$, and this cycle is a boundary. We define $D_q(M_j)(g_j) \in C'_{q+1}(M_j)$ to satisfy equation (†), use equation (b) to define $D_q(x)$ for all x, and complete the definition of D.

q.e.d.

$H^*(S^\infty \times_\pi X^2)$, $H_*(S^\infty \times_\pi X^2)$ の構造と Sq^* の定義

〜 $H_*(W \otimes_\pi H_*(X)^{(2)})$ と $H^*(\mathrm{Hom}_\pi(W, H^*(X)^{(2)}))$ における

cup product と cap product の確定と証明を中心として〜

R ; 可換体

π ; Z_2

$R\pi$, π の R 上の group ring

Def. group ring (群環)

π で生成される自由 R 加群において積を以下で定義

$$(r_1 + r_2 T)(r_1' + r_2' T) = (r_1 r_1' + r_2 r_2') + (r_1 r_2' + r_2 r_1')T$$

T は π の生成元

係数群はすべて R, $\otimes$ は $\otimes_R$ の略, $\otimes_\pi$ は $\otimes_{R\pi}$ の略, Hom は Hom_R の略, Hom_π は $\mathrm{Hom}_{R\pi}$ の略とする。

Def 1. W を次の i), ii) で定義される自由 π チェイン複体とする。

i) $q<0$ ならば $W_q = 0$ で, $q \geq 0$ ならば W_q はただ一つの元 w_q で生成される自由 π 加群

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