THE IMPACT OF PROCESS INNOVATION AND REAL BUSINESS CYCLES THEORY
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Abstract

The shockwaves from the September 15, 2008 Lehman Brothers collapse continue, with share prices still falling globally. Our position, however, is that the real economy is more important than financial markets. This paper addresses the cause of the current recession from a real-economy perspective. It does this with an analysis of real business cycles (RBC) theory using a new \( \log(\log(x + 1)) + 1 \)-type utility function rather than either the log or constant relative risk aversion (CRA) ones.

Using the RBC theory with a new utility function enables us to demonstrate that the progress of process innovation leads to a transition from supply to demand constraints in developed such economies as those in the United States and Japan, and that weak demand in those countries, even without demand saturation, has caused the current recession. Our use of this function has also led to such previously impossible findings as a strict formulation of involuntary unemployment and the existence of business cycles.

This paper demonstrates that utility functions’ shapes affect whether they find that progress in process innovation results in increased or decreased hours of labor. Furthermore, although both process and product innovation give companies advantages, their impact on overall economies are different.

Keywords: Process Innovation, Product Innovation, Real Business Cycles (RBC) Theory, Coefficient of Relative Risk Aversion

Introduction

The entire world has been experiencing a deep recession, the worst economic slowdown in Japan in 100 years, that many say began with the collapse of Lehman Brothers on September 15, 2008. In Japan, however, companies have continued to earn high profits, but the country itself has become an economically stratified society with a high unemployment rate, as in the United States. Almost no economists, including Keynesians, had predicted this deep a recession, but this phase of the business cycle has indeed taken place.

It is facile for economists to assert that the financial crisis caused this recession, and it is our contention that this is at best but a partial explanation. This paper therefore explains this recession’s cause theoretically using a model that addresses process innovation and proposes a recovery strategy for Japan and Japanese companies.

Many economists regard monetary phenomena as the cause of both the Great Depression and this recession. We, however, view the real economy as more important, as its factors underpin those monetary phenomena, and both can therefore fall into a vicious spiral.

Contemporary economics, however, has no theory explaining what causes the real economy to deteriorate. One contemporary framework, though, Kydland and Prescott’s real business cycles (RBC) theory (1), which is based on the work of Adam Smith (2) and Robert Lucas (3)(4), does explain business cycles from the perspective of the real economy. This paper therefore uses the RBC theory’s framework.

However, some empirical studies’ findings in regard to business cycles do not support RBC theory (5) (6). Our view, however, is that their explanation that falling productivity causes depressions is incorrect, particularly because they fail to explain why productivity falls. Instead, we see the problem as the capacity to supply becoming greater than demand, and using RBC theory’s framework we will demonstrate that economic downturns result from the overproduction that takes place in competitive market economies due to pressures resulting from the progress of process innovation.

This paper interprets conventional RBC theory by (a) considering the real economy to be important, (b) using the technique of the dynamic stochastic general equilibrium model, which makes such concepts as capital endogenous, (c) including such micro-foundations of macroeconomics as the utility function and the pain of labor, (d) selecting such real parameters as hours of labor as aggregators instead of money, and (e) using process innovation in a model.

Analyzing such concepts as capital as being endogenous based on dynamic general equilibrium theory, however, makes using the model to solve the problem particularly difficult. For this reason, we have conducted the model’s characteristic analysis using a simulation called calibration, avoiding a reality in which we cannot conduct a
characteristic analysis for a complicated utility function. The model uses either the log or constant relative risk aversion (CRRA) utility functions.

In view of the above, in order to use a more general utility function, such as $\log(\log(x+1)+1)$, we eliminated the chain of endogenous capital. Using this utility function enables us to explain changes in hours of labor due to the progress of process innovation and to explain the concepts of business cycles and involuntary unemployment. Finally, we position the current recession historically and propose a possible solution.

**Process Innovation and Product Innovation**

Innovations may generally be classified as either process or product innovations. Process innovations involve raising productivity by changing the process with which a firm makes its products. For example, a semiconductor factory may start producing more chips at less cost by making them smaller. An eighteenth-century pin factory being organized according to the division-of-work principle (2), and Ford’s development of conveyor-belt assembly lines are early examples of innovations that increased productivity per worker. Process innovations involve everyday work and tend to be easy to implement.

Product innovation involves developing completely new goods that change consumers’ lifestyles, such as what happened with televisions, washing machines, refrigerators, the Sony Walkman, personal computers, and the internet. When such products become popular with consumers entire new industries suddenly emerge, sometimes several in succession.

Process innovation and product innovation tend to happen simultaneously, and they have been largely responsible for the development of modern industrialized economies. Product innovation tends to happen first. If a new product becomes popular its developers employ more workers and increase production.

After a new product’s patents have expired, or other companies have found ways to produce similar products without violating the patents, process innovation becomes necessary in order to meet the competition. This results in a sudden rise in employment. When process innovation is performed positively, however, we cannot know whether employment increases or decreases. A growth industry is one that is still employing more workers, a mature industry is one in which employment is static, and in declining industries employment is decreasing.

This paper addresses how to distinguish growth industries from mature industries theoretically. Although it will later apply a specific model to this, we hypothesize that the present recession has resulted from progress in process innovation in such mature industries as the automotive sector in developed competitive economies, and that as a consequence of this their productive capacity has come to exceed demand, making automobile manufacturing a mature or declining industry, and resulting in large numbers of worker redundancies.

Samuelson stressed the fallacy of composition, which is “a fallacy in which what is a true of a part is, on that account alone, alleged to be also true of the whole” (7). For example, “If all farmers work hard and nature cooperates in producing a bumper crop, total farm income may fall, and probably will” (7). We agree with this and will employ the concept of Nash equilibrium to demonstrate it.

In market economies economic actors strive to maximize their individual utility instead of optimizing that of the entire system. Furthermore, even though people are both consumers and producers, as consumers they try to buy the best goods at the lowest possible prices and as producers they try to sell as many of their goods as possible at the highest possible prices, even if this means depriving others.

Conflicts of egos are common in competitive economies despite the presence of game theory’s Nash equilibrium. Societies based on the Nash equilibrium, however, are not inherently inferior to those based on total optimization, as the collapse of Communist regimes and the survival of capitalist ones indicates.

Some economists use game theory to study why this happened. They explain that capitalist incentives induce people to work hard, to make good products at low prices, and to devise still newer goods made more excellently than do socialist ones.

Competitive economies provide such incentives. Raising productivity greatly, however, may cause such economic downturns as the present recession and the Great Depression. Samuelson predicted what has been called the modern version of the Great Depression, caused by the overproduction of crops (7). This paper attempts to prove this prediction mathematically, based on RBC theory.

**Model**

This RBC model addresses goods that bring about capital accumulation extrinsically rather than endogenously. Its notations are $L$ for hours of labor, $A$ for extrinsic accumulation of capital due to increased productivity due to process innovation, $f(A,L)$ for the production function, $u(x)$.
for the utility function, and \( v(L) \) for the pain of labor. The first equation is:

\[
dA / dt = A > 0
\]  

(1)

for which \( A > 0 \). Then:

\[
\partial f(A,L) / \partial A = f_\alpha(A,L) > 0
\]  

(2)

Furthermore, \( f_\alpha(A,L) > 0 \) and \( f_{\alpha\alpha}(A,L) < 0 \), which means that the production function increases with \( A \), and the law of diminishing marginal productivity consists of \( L \). Next, \( u'(x) > 0 \) and \( u''(x) < 0 \), which means that the law of diminishing marginal utility holds here, \( x \) being the quantity of goods produced.

Since \( v'(L) > 0 \) and \( v''(L) \geq 0 \), when \( L \) increases the pain of labor increases more than proportionally. In market economies the market determines labor hours in order to maximize the whole utility, or \( \max u(f(A,L)) - v(L) \). This means that the first term, \( u(f(A,L)) \), is the utility of the quantity of goods produced by productivity and labor hours at time \( t \). The second term, \( v(L) \), is the pain of labor at that time. Market economies therefore select the hours of labor that maximize both the total utility of the goods produced and the pain of labor.

The next section explains why the conventional RBC theory includes the dilemma that the hours of labor is constant by assuming the log utility function.

**Hours of Labor With the Log Utility Function**

We assume here that the production function is, as usual, the Cobb-Douglas type (8). This means that:

\[
f(A,L) = AL^\gamma(0 < \gamma \leq 1)
\]  

(3)

We further assume that log includes a utility function. When we substitute these for the first utility function it becomes max log \((AL^\gamma) - v(L)\). When we differentiate this second utility function by hours of labor \( L \) the result is zero, or:

\[
\gamma L^{-1} - v'(L) = 0
\]  

(4)

This does not depend on variable \( \ell \) because \( A \) is absent, resulting in \( L \) remaining constant throughout time \( t \) and therefore not being dependent on changes in process innovation. This means that the utility function is selected badly (9).

We next investigate to find whether several utility functions exist. When time \( t \) and productivity \( A \) are small, increasing \( A \) leads to increased hours of labor \( L \), which is the equilibrium point of the utility of the goods produced and the pain of labor. When time \( t \) and productivity \( A \) are large, increasing \( A \) leads to decreases in the number of hours of labor \( L \) to the equilibrium point.

Moreover, the utility function \( u(x) \) has no ceiling, which means that it includes no demand saturation. If we can obtain such a utility function, we can explain that the number of hours of labor employed when the amount of goods produced increases suddenly and rapidly and that the number of hours of labor employed gradually decreases during a product or industry’s mature or declining periods.

Furthermore, if the utility function has no ceiling, the number of labor hours decreases even when the utility of the goods produced increases. This means that in market economies the number of hours of labor per person has downward rigidity (10), resulting in increased involuntary unemployment. We can therefore explain the phenomenon of the present recession by the existence of an outsider-insider. It is important for this paper to explain how such a utility function exists.

**The log(log(x + 1) + 1)-Type Utility Function**

This paper will here present the search for such an outsider-insider utility function. Differentiating the first utility function partially by hours \( L \) produces a result of zero.

\[
f_\alpha(A,L)u'(f(A,L)) - v'(L) = 0
\]  

(5)

In this equation \( v'(L) \) does not depend on productivity \( A \). The direction of the changes of productivity \( A \) on its left side therefore depends only on the equation’s first term. Differentiating this first term partially by productivity \( A \) produces the function \( \partial^2 u(f(A,L)) / \partial A \ell > 0 \).

If this unequal function is present, when \( A \) increases the equation’s first term becomes larger than its second term. The \( f_\alpha(A,L) \) of the first term therefore decreases the function of \( L \) because of the effects of the law of diminishing marginal production on \( L \). Next, \( u'(f(A,L)) \) is a decreasing function in regard to \( L \) because \( f(A,L) \) is an amplifying function in regard to \( L \) and the law of diminishing marginal utility applies. Furthermore, the \( v'(L) \) of the second term is an amplifying function in regard to \( L \) due to \( v''(L) \geq 0 \). When the first utility function is maximized, therefore, \( L \) increases.

If, however, the inequality \( \partial^2 u(f(A,L)) / \partial A \ell < 0 \) is present when the first utility function is maximized, \( L \) decreases. We can obtain the same
results by algebraic analysis if we differentiate equation (5) by time \( t \) and arrange for \( L \).

We further assume that \( f(A, L) \) includes a separation of the variables \( A \) and \( L \), as in the Cobb-Douglas production function of:

\[
Y = f(A, L) = AL^{\gamma} \quad (0 < \gamma \leq 1)
\]  

Thus, we assume that:

\[
f(A, L) = a(A)b(L)
\]  

In this paper we assume that all production functions are of this type.

This modifies the left side of the previous two functions in the following way:

\[
\partial^2 u f(A, L) / \partial A \partial L = \partial^2 (u(a(A)b(L))) / \partial A \partial L =
\partial (a(A)b(L))u(a(A)b(L)) / \partial L =
\alpha'(A)b'(L)u(a(A)b(L)) +
\alpha(A)b'(L)(u(a(A)b(L)) +
\alpha(A)b'(L)(u'(a(A)b(L)) + a(A)b'(L)u''(a(A)b(L))) -
\partial (a(A)b(L))u'(a(A)b(L)) / \partial L =
\alpha'(A)b'(L)d(u'(a(A)b(L))) / dx
\]  

Now let:

\[
x = a(A)b(L)
\]  

Then:

\[
\partial^2 u f(A, L) / \partial A \partial L = \alpha'(A)b'(L)u'(a(A)b(L)) + xu'(a(A)b(L)) =
\alpha'(A)b'(L)d(u'(a(A)b(L))) / dx
\]  

Subsequently, \( f_1 > 0, f_2 > 0 \) is present in \( \alpha'(A)b'(L) > 0 \). If the inequality \( \partial^2 u f(A, L) / \partial A \partial L < 0 \) is negative in regard to the function in equation (10), it is negative to the inequality \( u'(a(A)b(L)) + xu'(a(A)b(L)) < 0 \), and then from \( u'(a(A)b(L)) > 0 \) to \( xu'(a(A)b(L)) < 0 \).

The left side of this inequality is the coefficient of relative risk aversion (8). When it is greater than 1, as in the inequality \( xu'(a(A)b(L)) > 0 \), the inequality \( \partial^2 u f(A, L) / \partial A \partial L < 0 \) is present, and as a result the number of hours of labor present in market economies decreases in order to maximize the total utility of their goods produced and pain of labor.

We investigated the log(log(x + 1)) type utility function as one example of a utility function in which the coefficient of relative risk aversion changes from less than 1 to more than 1 as \( x \) increases. We calculate the differentiation of the first order of log(log(x + 1)) + 1, in which \( x \geq 0 \). Assuming that \( X = \log(x + 1) + 1 \) and differentiating it from \( x \), we obtained the first-order differentiation:

\[
d\log X / dx = (dX / dx)(d\log X / dX)
= (x + 1)^{\theta} \log(x + 1) + 1 > 0
\]  

We were, furthermore, able to obtain its of second-order differentiation from \( x \) with:

\[
-(x + 1)^{\theta} \log(x + 1) + 1 \leq 0
-\log(x + 1) - 2 < 0
\]  

This shows that we can therefore calculate the coefficient of relative risk aversion with:

\[
\frac{u'(x) + xu'(x)}{u(x)} = (x + 1)^{\theta} \log(x + 1) + 1 + x + 1
\]  

When \( x = 0 \) equation (13) is obviously equal to 1, which means that the coefficient of relative risk aversion is less than 1. Otherwise, when \( x \rightarrow +\infty \), equation (13) becomes less than 0, which makes the coefficient of relative risk aversion more than 1. Based on the intermediate value theorem, then, an \( x \) exists that satisfies \( \log(x + 1) + 1 = 0 \).

This means that this utility function has no ceiling.

However, the constant relative risk aversion (CRRRA) utility function’s, coefficient of relative risk aversion is always constant (8). It shows that the number of hours of labor is either always constant, in which case the CRRRA utility function is log, or it increases or decreases linearly, in which case business cycles do not exist.

Furthermore, when the CRRRA utility function’s coefficient of relative risk aversion is more than 1 it does have a ceiling. Equation (14), in which \( \theta > 1 \) is the coefficient of relative risk aversion, illustrates this.

\[
\lim_{x \rightarrow +\infty} u'(x) = \lim_{x \rightarrow +\infty} \log(x + 1) + 1 / (1 - \theta) \]  

Having a ceiling means that demand saturation is present. In reality, however, we often find that what happens is that sales continue to increase, prices and profits decrease, and workers lose their jobs. This section, then, has successfully explained demand saturation based on RBC theory.

**Numerical Example**

This chapter uses a numerical example to confirm the characteristics of the proposed utility function. We assume the utility function to be:

\[
u(x) = \log(x + 1) + 1
\]
The offset of 1 is added, as no essential difference is present. We next assume the pain of labor to be:

\[ v(L) = 0.3L \quad (16) \]

Finally, we assume the production function to be:

\[ x = f(A, L) = AL \quad (17) \]

We next analyzed how many hours of labor maximize total utility with:

\[ \max u(x) - v(L) = \max \log(\log(AL + 1) + 1) + 1 - 0.3L \quad (18) \]

We first maximized \( L \) by fixing \( t \). We then investigated the solution by changing time \( t \). Figure 1 illustrates the results of doing this. The productivity function \( A \) as the increasing function of \( t \) is the horizontal axis.

**Figure 1** Changes in hours labor and total utility over time

We differentiated the function in equation (18) as \( L \) and set it at 0, which is:

\[ A(AL + 1)^3(\log(AL + 1) + 1)^1 - 0.3 = 0 \quad (19) \]

We then differentiated equation (19) with time \( t \), setting \( L \) as 0, which is:

\[ A(AL + 1)^3(\log(AL + 1) + 1) - AAL = 0 \quad (20) \]

Reducing equation (20) produces:

\[ \log(AL + 1) + 1 - AL = 0 \quad (21) \]

Combining equations (19) and (21) produces:

\[ A(AL + 1)^3(AL)^1 - 0.3 = 0 \quad (22) \]

Reducing equation (22) produces:

\[ L = 1/(0.3(AL + 1)) \quad (23) \]

Equation (21) enables us to calculate \( AL \), substitute the \( AL \) that we obtained in equation (23), and then calculate \( L \), which enables us to discover \( A \), which enables us to find that \( AL = 2.1462 \), that \( L = 1.0595 \), and that \( A = 2.0257 \), which is the maximum amount of hours of labor \( L \) over the time period \( t \).

This section has explained how the number of hours of labor can decrease without demand saturation, even though at the same time utility increases due to the progress of process innovation.

**Conclusion**

In order to show the influences of process innovation that existing economic theory had not explained, this paper presented a new utility function to replace that of CRRA, which many such models as RBC use but which has the limitation of being unable to explain business cycles. In order to do this we have proposed the \( \log(\log(x + 1) + 1) \)-type utility function instead.

This function can explain the phenomenon of the utility of goods increasing while the hours of labor decrease. Because its coefficient of relative risk aversion is less than 1 at first and equal to 1 and more than 1 when \( x \) increases, it explains how the phenomenon occurs without demand saturation. This also explains the mechanism through which involuntary unemployment and business cycles result from progress in process innovation due to market economies having downward rigidity of hours of labor per person.

This paper’s importance is that it considers the case of the present recession in the light of the Great Depression that began in 1929. This could be said to have resulted from the agrarian crisis, which occurred when the entire world shifted to an industrial society from an agrarian one.

The present recession is the result of progress in process innovation while such established industries as the auto industry in such advanced countries with competitive societies as the United States and Japan entered periods of maturity or decline. Even though the mathematical coefficient of relative risk aversion within such industries has become more than 1, they still actively compete in the global market.

The general capitalist answer for how to address this recession is that entrepreneurs need to start up new industries (11). The United States’ Obama Administration’s Green New Deal policy is an example of this. If such entrepreneurs do not appear in societies that aim for Nash equilibrium, we also need such policies directed at overall optimization as work-sharing.

Furthermore, developed nations introducing advanced technologies into developing ones,
where the coefficient of relative risk aversion is smaller than 1, may be one method of increasing demand in the developing world. This follows the argument that demand in China aims can help global recovery from the present recession.

References